A New Three Object Triangulation Algorithm for Mobile Robot Positioning

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Abstract

Positioning is a fundamental issue in mobile robot applications. It has to be performed in many tasks. Among them, triangulation based on angles measurements between three points is a classical method. Many papers have been published on this topic. Unfortunately, some aspects are not addressed. For example, some of them are qualified as barycentric between landmarks, have limited ranges, or only work within the triangle defined by the three beacons. Moreover, to the best of our knowledge, no paper has been devoted to the computation of the uncertainties associated to the triangulation method.

In this paper, we present a new three and two dimensional triangulation method, named 3-DTS, that relaxes some of the above-cited assumptions and lacks of knowledge. We also provide a comprehensive comparison between many algorithms, and show that our algorithm is faster and more accurate than most of the others. Finally, we show that our method is well-suited for mobile robot applications because it allows to compute the uncertainties of its outputs, which can be used to identify outlier measurements, or as a validation gate in Kalman filters.

1 Introduction

Positioning is a fundamental issue in mobile robot applications. Indeed, in many tasks, a mobile robot that moves in an environment has to position itself with respect to other objects. Among the many methods that have been published to address this problem, triangulation based on angles measurements between three points is a classical method. Many papers have been published on this topic. Unfortunately, some aspects are not addressed. For example, some of them are qualified as barycentric between landmarks, have limited ranges, or only work within the triangle defined by the three beacons. Moreover, to the best of our knowledge, no paper has been devoted to the computation of the uncertainties associated to the triangulation method. In other words, the reliability of the computed robot position is not known, which is a major drawback for many applications, such as mobile robot navigation or localization in multiform. An alternative positioning method is based on trilateration, which requires distance measurements.

In this paper, we propose a new three and two dimensional triangulation method, named 3-DTS, that relaxes some of the above-cited assumptions and lacks of knowledge. We also provide a comprehensive comparison between many algorithms, and show that our algorithm is faster and more accurate than most of the others. Finally, we show that our method is well-suited for mobile robot applications because it allows to compute the uncertainties of its outputs, which can be used to identify outlier measurements, or as a validation gate in Kalman filters.

2 Related Work

2.1 Triangulation algorithms

The principle of triangulation has seveded to many engines, and many methods have been proposed so far. One of the first comprehensive surveying work has been published by Hallert [1] in 1961. Since this paper, many others have been published. For instance, Lee [2] presented a method for the three dimensional case, Sutherland and Hodgman [3] proposed a polygon clipping algorithm, Chen [4] and Betke and Nishihara, Papoulis, Chandran and Kenefick [5] published papers about the minimum error in triangulation, Hung [6] presented a linear triangulation method taking into account uncertainties, Stroupe and Balch [7] proposed a probabilistic method for multi-robot cooperative localization, Olson [8] proposed a fast analytical triangulation method based on the triangle median intersection, Lecluse et al. [9] proposed a triangulation algorithm using polar coordinates, Esteban and Nishihara [10] proposed a sonar based triangulation method, Madhavan and Durrant-Whyte [11] proposed a natural landmark based localization method using laser range finder and GPS measurements, Goldberg et al. [12] described a robust exploration and mapping method based on stereo vision, Olson [13] proposed probabilistic and visual landmarks for localization purposes, Cobzas et al. [14] described panoramic vision based localization and modeling, Baltzakis et al. [15] proposed a hybrid framework for fast visual checks of physical objects using stereo pairs and laser range-finders, Emanuele et al. [16] described a 3D hybrid framework for occupancy grid based mapping and localization for rescue robots, Pathak et al. [17] described uncertainty analysis for optic flow based visual odometry, Parra et al. [18] described trinocular ground plane estimation, Burguera et al. [19] proposed a robust method based on EKF and UKF, Sola et al. [20] proposed a fast vision only EKF SLAM method, Marzorati et al. [21] proposed a combination of particle filters and Delaunay triangulation for SLAM, Bennewitz et al. [22] described a hybrid metric-topo-metric approach for large environments, Frese [23] described a multilevel relaxation for simultaneous localization and mapping, Newman et al. [24] described a 3D laser based SLAM, Nuchter et al. [25] described 6D SLAM with an application to autonomous mine mapping, Lim et al. [26] described an appearance based method for large-scale SLAM, Agrawal et al. [27] described censure: a fast detector for visual SLAM, etc. Most of these papers deal with sensors, or with SLAM, but only few of them focus on the triangulation method itself. Indeed, most of the papers that deal with triangulation only apply the triangulation method without any improvement. Among the few papers that focus on triangulation, Betke et al. [28] formulated the triangulation problem as a weighted sum of least squares problem, Stroupe et al. [29] proposed a probabilistic extension of the triangulation method, Sutherland and Hodgman [3] proposed a polygon clipping algorithm, Olson [30] proposed a probabilistic formulation for the triangulation problem, Cohen et al. [31] proposed a method which turns out to be exactly the same as the one proposed by Betke et al. [28], Madhavan et al. [32] proposed a triangulation method based on the median intersection of three circles, Esteban et al. [33] proposed a triangulation method based on a probabilistic model of the sensors, Lecluse et al. [34] proposed a triangulation method based on polar coordinates, Zwick et al. [35] and Kahn et al. [36] proposed a fast triangulation method based on a linear formulation, Stroupe et al. [37] proposed a method which turns out to be exactly the same as the one proposed by Betke et al. [28], Madhavan et al. [38] proposed a probabilistic triangulation method based on particle filters, Sola et al. [39] proposed a fast triangulation method based on a linear formulation, Lamon et al. [40] proposed a triangulation method based on the geometrical properties of the triangle formed by the three beacons, Gonçalves et al. [41] proposed a method which turns out to be exactly the same as the one proposed by Betke et al. [28], Sola et al. [42] proposed a fast triangulation method based on a linear formulation, Sola et al. [43] proposed a fast triangulation method based on a linear formulation, Zhang et al. [44] proposed a method which turns out to be exactly the same as the one proposed by Betke et al. [28], Gonçalves et al. [45] proposed a fast triangulation method based on a linear formulation, Sola et al. [46] proposed a fast triangulation method based on a linear formulation, Sola et al. [47] proposed a fast triangulation method based on a linear formulation, Sola et al.

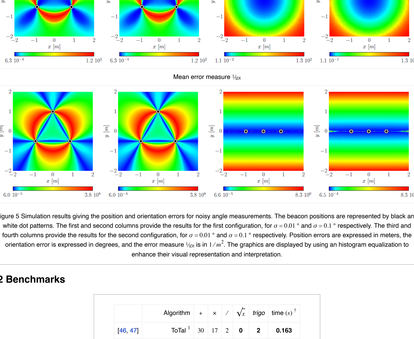

Figure 5 Simulation results giving the position and orientation errors for noisy angle measurements. The beacon positions are represented by black and white dot patterns. The first and second columns provide the results for the first configuration, for $\alpha = 0.01^\circ$ and $\sigma = 0.1^\circ$ respectively. The third and fourth columns provide the results for the second configuration, for $\alpha = 0.01^\circ$ and $\sigma = 0.1^\circ$ respectively. Position errors are expressed in meters, the orientation error is expressed in degrees, and the error measure $\bar{\epsilon}_{\mathbb{P}X}$ is in $1/m^2$. The graphics are displayed by using an histogram equalization to enhance their visual representation and interpretation.

4.2 Benchmarks

	Algorithm	+	×	$\sqrt{\cdot}$	trigo	time (s) [†]
[46, 47]	ToTal [‡]	30	17	2	**0**	**2**	**0.163**
[38]	Ligas [‡]	29	22	2	**0**	**2**	0.171
[29]	Font-Llagunes [‡]	23	17	2	**0**	5	0.223
[42]	Cassini [‡]	**19**	**8**	3	**0**	4	0.249
[5]	Cohen [‡]	37	15	3	2	4	0.272
[2]	Easton [‡]	22	24	**1**	**0**	5	0.298
[9]	McGillem [‡]	37	18	5	2	8	0.340
[17]	Hmam [‡]	29	11	3	3	9	0.428
[5]	Cohen [‡]	26	11	3	2	11	0.437
[34]	Esteves [‡]	43	14	2	2	11	0.471
[42]	Collins [‡]	34	10	2	2	11	0.485
[9]	McGillem [‡]	29	9	3	2	11	0.501
[42]	Kaestner [‡]	28	10	3	2	11	0.504
[45]	Tsukiyama [‡]	52	22	3	5	14	0.596
[16]	Casanova [‡]	52	21	4	5	14	0.609
[32]	Tienstra [‡]	33	18	8	3	9	0.640
[31]	Font-Llagunes [‡]	62	25	6	1	8	0.648
[8]	Madsen [‡]	38	24	5	3	15	0.707

[†] For 10^6 executions on an Intel(R) Core(TM) i7 920 @ 2.67GHz.
[‡] Geometric circle intersection [‡] Trigonometric solution

Table 1 Comparison of various triangulation algorithms to our *ToTal* algorithm.

We have also compared the execution time of our algorithm to 17 other three object triangulation algorithms similar to ours (*i. e.* which work in the whole plane and for any beacon ordering). These algorithms have been introduced in Section 21, and have been implemented after the author's guidelines[21]. Each algorithm has been running 10^6 times at random locations of the same square shaped area as in the error analysis. The last column of Table 11 provides the running times on an *Intel(R) Core(TM) i7 920 @ 2.67GHz* (6GB RAM, Ubuntu 11.04, GCC 4.5.2). We used the C clock_gettime function to measure the execution times, in order to yield reliable results under timesharing. It appears that our algorithm is the fastest of all (about 30 % faster than the last best known algorithm of Font-Llagunes [29], and 5 % faster than the recent algorithm of Ligas [38]). In addition to the computation times, we have also reported the number of basic arithmetic computations, squared roots, and trigonometric functions used for each algorithm. This may help to choose an algorithm for a particular hardware architecture, which may have a different behavior for basic arithmetic computations, or complex functions such as square root or trigonometric functions. One can see that our algorithm has the minimum number of trigonometric functions, which is clearly related to the times on a classical computer architecture (see Table 11). A fast algorithm is an advantage for error simulations, beacon placement, and beacon position optimization algorithms. Note that the algorithm of Ligas also uses the minimum number of trigonometric functions (two $\cos(.)$ computations) like *ToTal*, explaining why both algorithms are basically similar in terms of efficiency. However, the algorithm of Ligas does not provide a reliability measure, contrarily to our algorithm *ToTal*.

5 Conclusions

Most of the many triangulation algorithms proposed so far have major limitations. For example, some of them need a particular beacon ordering, have blind spots, or only work within the triangle defined by the three beacons. More reliable methods exist, but they have an increasing complexity or they require to handle certain spatial arrangements separately.

This paper presents a new three object triangulation algorithm based on the elegant notion of power center of three circles. Our new triangulation algorithm, which is called *ToTal*, *natively* works in the whole plane (except when the beacons and the robot are concyclic or collinear), and for any beacon ordering. Furthermore, it only uses basic arithmetic computations and two $\cos(.)$ computations. Comprehensive benchmarks show that our algorithm is faster than comparable algorithms, and simpler in terms of the number of operations. In this paper, we have compared the number of basic arithmetic computations, squared root, and trigonometric functions used for 17 known triangulation algorithms.

In addition, we have proposed a unique reliability measure of the triangulation result in the whole plane, and established by simulations that $\bar{\epsilon}_{\mathbb{P}X}$ is a natural and adequate criterion to estimate the error of the positioning. To our knowledge, none of the algorithms of the same family does provide such a measure. This error measure can be used to identify the pathological cases (critic circumference), or as a validation gate in Kalman filters based on triangulation.

For all these reasons, *ToTal* is a fast, flexible, and reliable three object triangulation algorithm. Such an algorithm is an excellent choice for many triangulation issues related to the performance or optimization, such as error simulations, beacon placement or beacon position optimization algorithms. It can also be used to understand the sensitivity of triangulation with respect to the the input angles. A fast and inexpensive algorithm is also an asset to initialize a positioning algorithm that internally relies on a Kalman filter.

Appendix

In this section, we detail the sensitivity analysis of the computed position. We start by computing the derivative of x_R with respect to the first angle ϕ_1

$$\begin{aligned}\frac{\partial x_R}{\partial \phi_1} &= \frac{\partial K_{12}}{\partial \phi_1}(y'_{12}-y'_{13}) + \frac{\partial K_{13}}{\partial \phi_1}(y'_{13}-y'_{12})\frac{K_{12}}{D} \\ &\quad + \frac{\partial\left(\frac{1}{D}\right)}{\partial \phi_1}K_{12}(y'_{12}-y'_{13}) \\ &= \frac{a_1(.)}{D} + \frac{a_2(.)}{D} - \frac{d_1(.)}{D^2} - \frac{1}{D}g_1(.),\end{aligned} \tag{30}$$

where $g_1(.)$ is some function of all the other parameters. Similar results yield for the derivative of x_R with respect to the second and third angles, ϕ_2 and ϕ_3 respectively

$$\begin{aligned}\frac{\partial x_R}{\partial \phi_2} &= \frac{1}{D}g_2(.), \quad \frac{\partial x_R}{\partial \phi_3} = \frac{1}{D}g_3(.),\end{aligned} \tag{31}$$

where $g_2(.)$ and $g_3(.)$ are some functions of all the other parameters. The total differential of x_R with respect to ϕ_1, ϕ_2, and ϕ_3 is given by

$$\begin{aligned}\Delta x_R &= \frac{\partial x_R}{\partial \phi_1}\Delta\phi_1 + \frac{\partial x_R}{\partial \phi_2}\Delta\phi_2 + \frac{\partial x_R}{\partial \phi_3}\Delta\phi_3 \\ &= \frac{1}{D}\Delta\phi\left(g_1(.)+g_2(.)+g_3(.)\right) \\ &= \frac{1}{D}\Delta\phi\; g(.),\end{aligned} \tag{32}$$

where we assumed that the three infinitesimal increments are equal $\Delta\phi = \Delta\phi_1 = \Delta\phi_2 = \Delta\phi_3$. A similar result yields for the total differential of y_R

$$\Delta y_R = \frac{1}{D}\Delta\phi\; h(.) \tag{33}$$

where $h(.)$ is some function of all the other parameters. Finally, we can compute Δd_R as follows

$$\begin{aligned}\Delta d_R &= \sqrt{(\Delta x_R)^2 + (\Delta y_R)^2} \\ &= \frac{1}{|D|}|\Delta\phi|\sqrt{g(.)^2 + h(.)^2} \\ &= \frac{1}{|D|}|\Delta\phi|\, f(.)\end{aligned} \tag{34}$$

where $f(.)$ is some function of all the other parameters.

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